POISSON DISTRIBUTION

The calculations for a binomial distribution are very cumbersome for large values of n. If n is large while the probability p of occurrence of an event is close to zero, the event is called a *rare event*. Some examples of such situations in daily life are traffic problems with repeated occurrences of events such as accidents whose probability is very small, probability of events such as faults and breakdowns in industry, typographical errors in a book, and so on. In such cases, the binomial distribution is closely approximated by another discrete distribution called Poisson distribution, which is much more easier to compute. The Poisson distribution is the limiting form of Binomial distribution when number of trials, n is very large ($n \to \infty$) and probability of success is small ($p \to 0$) such that np always remains finite, say λ. In practice, if $n \geq 50$ and $\lambda = np \leq 5$, the Poisson distribution gives good approximation of a Binomial distribution.

POISSON DISTRIBUTION AS A LIMITING FORM OF BINOMIAL DISTRIBUTION

Now we will derive the Poisson approximation to the binomial distribution by assuming that $n \to \infty$ and $p \to 0$ such that the product np always remains finite, say λ. We will use a result from calculus that

$$\underset{x \to \pm\infty}{\text{Lt}} \left(1 + \frac{1}{x}\right)^x = e,$$

where e is a constant lying between 2 and 3, and e^x is defined by

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots \text{ to } \infty,$$

where x is a real number.

Now $np = \lambda \Rightarrow p = \dfrac{\lambda}{n}$.

From binomial distribution, we have

$$P(r) = P(X = r) = {}^nC_r\, p^r q^{n-r} \qquad (\text{for } r = 0, 1, 2, \ldots, n)$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} \cdot \frac{\lambda^r}{n^r}\left(1 - \frac{\lambda}{n}\right)^{n-r} \qquad \left(\text{Putting } p = \frac{\lambda}{n}, q = 1 - p = 1 - \frac{\lambda}{n}\right)$$

$$= \frac{\lambda^r}{r!}\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\ldots\left(1 - \frac{r-1}{n}\right) \cdot \frac{\left(1 - \frac{\lambda}{n}\right)^n}{\left(1 - \frac{\lambda}{n}\right)^r} \qquad \ldots(i)$$

When $n \to \infty$, each of the factors $1 - \frac{1}{n}, 1 - \frac{2}{n}, \ldots 1 - \frac{r-1}{n}$ tends to 1; $\left(1 - \frac{\lambda}{n}\right)^r$ also tends to 1 as λ and r are finite, and

$$\underset{n \to \infty}{\text{Lt}} \left(1 - \frac{\lambda}{n}\right)^n = \underset{\frac{n}{\lambda} \to \infty}{\text{Lt}} \left[\left(1 - \frac{\lambda}{n}\right)^{-\frac{n}{\lambda}}\right]^{-\lambda} = e^{-\lambda}$$

Hence, from (i), we get

$$\underset{n \to \infty}{\text{Lt}}\ P(r) = \frac{\lambda^r}{r!} e^{-\lambda}$$

Hence in the limiting case, when $n \to \infty, p \to 0$, such that $np = \lambda$ = a finite number, the binomial distribution reduces to the Poisson distribution with probability function $P(r)$ defined by

$$\mathbf{P(r) = P(x = r) = \frac{e^{-\lambda}\lambda^r}{r!}, r = 0, 1, 2, \ldots \text{ to } \infty}$$

Let us assure ourselves that this limiting function is indeed a probability function:

$$\sum_{r=0}^{\infty} P(r) = e^{-\lambda} + \frac{\lambda e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!} + \frac{\lambda^3 e^{-\lambda}}{3!} + \ldots \text{ to } \infty$$

$$= e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots \text{ to } \infty\right)$$

$$= e^{-\lambda} . e^{\lambda} = 1.$$

Note that binomial distribution has two parameters n and p, but Poisson distribution has only one parameter λ. Tables are available to find the values of $e^{-\lambda}$, so calculations are easy.

RECURRENCE FORMULA FOR PROBABILITIES OF POISSON DISTRIBUTION

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} \text{ and } P(r+1) = \frac{e^{-\lambda}.\lambda^{r+1}}{(r+1)!}$$

$$\frac{P(r+1)}{P(r)} = \frac{e^{-\lambda}.\lambda^{r+1}}{(r+1)!}\cdot\frac{r!}{e^{-\lambda}\lambda^r} = \frac{\lambda}{r+1}$$

Thus $P(r+1) = \frac{\lambda}{r+1} P(r)$, for $r = 0, 1, 2, 3, \ldots$

Hence to find successive probabilities, we calculate $P(0) = e^{-\lambda}$, and then use

$$P(1) = \lambda\, P(0),\ P(2) = \frac{\lambda}{2}\, P(1),\ P(3) = \frac{\lambda}{3}\, P(2), \ldots$$

MEAN AND VARIANCE OF POISSON DISTRIBUTION

For the Poisson distribution, we have

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} \quad \text{for } r = 0, 1, 2, \ldots$$

$$\therefore \quad E(X) = \mu = \sum_{r=0}^{\infty} r\,P(r) = \sum_{r=0}^{\infty} r\frac{e^{-\lambda}\lambda^r}{r!}$$

$$= 0 + 1.\lambda e^{-\lambda} + 2\frac{\lambda^2 e^{-\lambda}}{2!} + 3\frac{\lambda^3 e^{-\lambda}}{3!} + \ldots \text{ to } \infty$$

$$= \lambda e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots \text{ to } \infty\right) = \lambda e^{-\lambda}.e^{\lambda} = \lambda.$$

Thus the **mean of the Poisson distribution is equal to the parameter λ.**

The variance σ^2 is given by

$$\sigma^2 = \sum_{r=0}^{\infty} r^2\,P(r) - \mu^2 = \sum_{r=0}^{\infty} r^2\frac{e^{-\lambda}\lambda^r}{r!} - \lambda^2$$

$$= e^{-\lambda}\left[1^2.\frac{\lambda}{1!} + 2^2.\frac{\lambda^2}{2!} + 3^2.\frac{\lambda^3}{3!} + 4^2.\frac{\lambda^4}{4!} \ldots \text{ to } \infty\right] - \lambda^2$$

$$= \lambda e^{-\lambda}\left[1 + \frac{2\lambda}{1!} + \frac{3\lambda^2}{2!} + \frac{4\lambda^3}{3!} + \ldots \text{ to } \infty\right] - \lambda^2$$

$$= \lambda e^{-\lambda}\left[1 + \frac{(1+1)\lambda}{1!} + \frac{(1+2)\lambda^2}{2!} + \frac{(1+3)\lambda^3}{3!} + \ldots \text{ to } \infty\right] - \lambda^2$$

$$= \lambda e^{-\lambda}.\left[\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots \text{ to } \infty\right) + \lambda\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots \text{ to } \infty\right)\right] - \lambda^2$$

$$= \lambda e^{-\lambda}\,(e^{\lambda} + \lambda e^{\lambda}) - \lambda^2 = \lambda + \lambda^2 - \lambda^2 = \lambda.$$

Hence the **variance of the Poisson distribution is also λ, and standard deviation is** $\sqrt{\lambda}$.

Poisson distribution is always positively skewed (long tail towards the right–makes sense as we are talking about rare events).

ILLUSTRATIVE EXAMPLES

Example 1. *For a Poisson distribution, find*

(i) $P(2)$, *given* $\lambda = 1$ *(ii)* $P(3)$, *given* $\lambda = \frac{1}{2}$.

Solution. For a Poisson distribution,

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

(i) $P(2) = \frac{1^2 . e^{-1}}{2!} = \frac{0.368}{2}$ [$\because e^{-1} = 0.368$, from table]

$= 0.184$

(ii) $P(3) = \frac{\left(\frac{1}{2}\right)^3 . e^{-0.5}}{3!} = \frac{\frac{1}{8} \times 0.607}{6}$ [$\because e^{-0.5} = 0.607$, from table]

$= \frac{0.607}{48} = 0.013$

Example 2. *If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals (i) exactly 3, (ii) more than 2 individuals will suffer from a bad reaction. (Use* $e^{-2} = 0.1353$*)*

Solution. Let X be the number of individuals suffering a bad reaction. Then X has a binomial distribution with $n = 2000$, $p = 0.001$. However, the calculations would be very laborious using binomial distribution. Hence we can use Poisson approximation with $\lambda = np = (2000)(0.001) = 2$.

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!} = \frac{2^r e^{-2}}{r!}$$

(i) $P(X = 3) = \frac{2^3 e^{-2}}{3!} = \frac{8}{6} e^{-2} = \frac{4}{3}(0.1353) = 0.180$

(ii) $P(X > 2) = 1 - [P(0) + P(1) + P(2)] = 1 - \left[\frac{2^0 e^{-2}}{0!} + \frac{2^1 e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!}\right]$

$= 1 - 5e^{-2} = 1 - 5(0.1353) = 0.323$

Example 3. *In a certain factory turning razor blades, there is a small chance* $\frac{1}{500}$ *for any blade to be defective. The blades are in packets of 10. Use Poisson's distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10000 packets. (Use* $e^{-0.02} = 0.9802$*)*

Solution. Here $N = 10000$, $n = 10$, $p = \frac{1}{500} = 0.002$

$\Rightarrow \lambda = np = 10 \times 0.002 \Rightarrow \lambda = 0.02$

P (No defective blade) $= P(X = 0) = \frac{e^{-0.02}(0.02)^0}{0!}$

$= e^{-0.02} = 0.9802$

$\therefore$ The approximate number of packets containing no defective blade

$= 10000 \times 0.9802 = 9802$

$$P(\text{one defective blade}) = P(X = 1) = \frac{e^{-0.02}(0.02)^1}{1!}$$

$$= 0.9802 \times 0.02 = 0.0196$$

$\therefore$ The approximate number of packets containing one defective blade = $10000 \times 0.0196 = 196$

$$P(\text{two defective blades}) = P(X = 2) = \frac{e^{-0.02}(0.02)^2}{2!} = \frac{0.9802 \times 0.0004}{2}$$

$$= 0.000196 = 0.0002$$

$\therefore$ The approximate number of packets containing two defective blades = $10000 \times 0.0002 = 2$.

Example 4. *A car hire firm has two cars, which it hires out day by day. The number of demands for cars on each day is distributed as a Poisson distribution with mean 1.5. Calculate the probabilities of days on which neither car is used and the probabilities of days on which some demand is refused.* (*Use* $e^{-1.5} = 0.2231$)

Solution. Here λ = mean = 1.5

The probability of days when no car is required

$= P(0) = e^{-\lambda} = e^{-1.5} = 0.2231$

The probability of days when some demand is refused

= Prob. (more than 2 cars are demanded) = $1 - [P(0) + P(1) + P(2)]$

$$= 1 - e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!}\right) = 1 - 0.2231\,(1 + 1.5 + 1.125) = 0.1913$$

Example 5. *Suppose that X has a Poisson distribution. If* $P(X = 2) = \frac{2}{3}P(X = 1)$, *evaluate* $P(X = 0)$.

Solution. Given $P(X = 2) = \frac{2}{3}P(X = 1)$

$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3}\frac{e^{-\lambda}\lambda}{1!} \Rightarrow \lambda = \frac{4}{3}$$

Hence, $P(X = 0) = e^{-\lambda} = e^{-4/3}$.

Example 6. *Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective, by using (i) the binomial distribution (ii) Poisson distribution.*

Solution. Here $n = 10$, $p = \frac{10}{100} = 0.1$, so $\lambda = np = (10)(0.1) = 1$.

(*i*) $P(X = 2) = {}^nC_2 p^2 q^{n-2} = {}^{10}C_2 p^2 q^8 = \frac{10.9}{1.2}(0.1)^2(0.9)^8 = 0.1937$

(*ii*) $P(X = 2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-1}1^2}{2} = \frac{1}{2}e^{-1} = \frac{1}{2}(0.368) = 0.184$ [Using table]

Remark

Thus, in the above example, we see that the approximation is quite good. For $n \geq 50$ and $np \leq 5$, we get extremely close results.

Example 7. *If the variance of the Poisson distribution is 2, find the probabilities for r = 1, 2, 3, 4 and 5 from the recurrence relation of the Poisson distribution.* (*Use* $e^{-2} = 0.1353$)

Solution. As the variance is λ, we get $\lambda = 2$.

$P(0) = e^{-\lambda} = e^{-2} = 0.1353.$

Now we use the recurrence relation of the Poisson distribution

$$P(r+1) = \frac{\lambda}{r+1} P(r)$$

$$P(1) = \frac{\lambda}{1} P(0) = 2\ (0.1353) = 0.2706$$

$$P(2) = \frac{\lambda}{2} P(1) = P(1) = 0.2706$$

$$P(3) = \frac{\lambda}{3} P(2) = \frac{2}{3} (0.2706) = 0.1804$$

$$P(4) = \frac{\lambda}{4} P(3) = \frac{1}{2} (0.1804) = 0.0902$$

$$P(5) = \frac{\lambda}{5} P(4) = \frac{2}{5} (0.0902) = 0.0361$$

Example 8. *A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, what is the probability that*

(a) 2 or less bicycle riders will use the cycle track within an hour?

(b) 3 or more bicycle riders will use the cycle track within an hour?

Also, write the mean expectation and variance for the random variable X.

Solution. Given mean = $\lambda = 3.2$

Let X be the number of bicycle riders which use the cycle track.

(*a*) Required probability = $P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{e^{-3.2}(3.2)^0}{\underline{|0}} + \frac{e^{-3.2}(3.2)^1}{\underline{|1}} + \frac{e^{-3.2}(3.2)^2}{\underline{|2}}$$

$$= e^{-3.2}\ [1 + 3.2 + 5.12]$$

$$= 0.041 \times 9.32 = 0.382$$

(*b*) Required probability = $P(X \ge 3) = 1 - P(X \le 2)$

$$= 1 - 0.382 = 0.618$$

Also, mean expectation and variance of X are $\lambda = 3.2$

Example 9. *A particular river near a small town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation? Also, calculate the probability of 3 or less overflow floods in a 10 years interval.*

Solution. Given the average event of flood overflow in every 10 years is 2.

So, $\lambda = 2$.

$\therefore$ Required probability = $P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{2^0 e^{-2}}{\underline{|0}} + \frac{2^1 e^{-2}}{\underline{|1}} + \frac{2^2 e^{-2}}{\underline{|2}} + \frac{2^3 e^{-2}}{\underline{|3}}$$

$$= e^{-2}\left(1 + 2 + 2 + \frac{4}{3}\right)$$

$$= 0.135 \times \frac{19}{3} = 0.045 \times 19$$

$$= 0.855 = 0.86$$

Example 10. *For a Poisson distribution model, if arrival rate of passengers at an airport is recorded 30 per hour on a given day. Find*

(a) *the expected number of arrivals in the first 10 minutes of an hour.*

(b) *the probability of exactly 4 arrivals in the first 10 minutes of an hour.*

(c) *the probability of 4 or fewer arrivals in the first 10 minutes of an hour.*

(d) *the probability of 10 or more arrivals in an hour given that there are 8 arrivals in the first 10 minutes of that hour.*

Solution. (*a*) Given number of arrivals per hour is 30.

Let the random variable X be the number of arrivals in first 10 minutes of an hour *i.e.* $\frac{1}{6}$ hour.

$\therefore \quad E(X) = 30 \times \frac{1}{6} = 5.$

Hence, the expected number of arrivals in the first 10 minutes of an hour is 5.

(*b*) For first 10 minutes of an hour $\lambda = 5$.

$\therefore$ P(exactly 4 arrivals in first 10 minutes of an hour)

$$= \frac{5^4 e^{-5}}{\lfloor 4} = \frac{625 \times 0.0067}{24} = 0.174$$

(*c*) P(4 or fewer arrivals in first 10 minutes of an hour)

$$= P(X \le 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$$

$$= \frac{5^0 e^{-5}}{\lfloor 0} + \frac{5^1 e^{-5}}{\lfloor 1} + \frac{5^2 e^{-5}}{\lfloor 2} + \frac{5^3 e^{-5}}{\lfloor 3} + \frac{5^4 e^{-5}}{\lfloor 4}$$

$$= e^{-5}\left[1 + 5 + \frac{25}{2} + \frac{125}{6} + \frac{625}{24}\right] = 0.0067 \times \frac{1569}{24} = 0.438$$

(*d*) Given that number of arrivals in first 10 minutes is 8.

So, expected number of arrivals in next 50 minutes *i.e.* $\frac{5}{6}$ hour is $30 \times \frac{5}{6} = 25$.

So, for next 50 minutes, $\lambda = 25$.

Now, P(10 or more arrivals in an hour given that there are 8 arrivals in the first 10 minutes of that hour)

$$= P(X \ge 2) = 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[\frac{25^0 e^{-25}}{\lfloor 0} + \frac{25^1 e^{-25}}{\lfloor 1}\right] = 1 - 26e^{-25}.$$

Example 11. *Phone calls arrive at the rate of 48 per hour at the reservation desk for Indian Airlines.*

(*i*) *Compute the probability of receiving three calls in a 5 minutes interval of time.*

(*ii*) *Compute the probability of receiving exactly 10 calls in 15 minutes.*

(*iii*) *Suppose no calls are currently on hold. If the agent takes 5 minutes to complete the current call, how many do you expect to be waiting by that time? What is the probability that none will be waiting?*

(*iv*) *If no calls are currently being processed, what is the probability that the agent can take 3 minutes for personal time without being interrupted by a call?*

Solution. Given number of phone calls arrives per hour is 48.

(*i*) Let random variable X be the number of phone calls in 5 minutes interval of time, then

$$\lambda = 48 \times \frac{5}{60} \Rightarrow \lambda = 4.$$

$$\text{So, } P(X = 3) = \frac{4^3 . e^{-4}}{\lfloor 3} = \frac{64 \times 0.018}{6} = 0.192$$

(ii) Let random variable X be the number of phone calls in 15 minutes interval of time, then

$$\lambda = 48 \times \frac{15}{60} \Rightarrow \lambda = 12.$$

So, $P(X = 10) = \dfrac{12^{10} e^{-12}}{\lfloor 10}$.

(iii) Let random variable X be the number of phone calls in 5 minutes interval of time, then

$$\lambda = 48 \times \frac{5}{60} \Rightarrow \lambda = 4.$$

So, we expect that 4 callers will be waiting during that time.

Now, $P(X = 0) = \dfrac{4^0 e^{-4}}{\lfloor 0} = 0.018$

(iv) Let random variable X be the number of phone calls in 3 minutes interval of time, then

$$\lambda = 48 \times \frac{3}{60} \Rightarrow \lambda = 2.4$$

$$P(X = 0) = \frac{2.4^0 e^{-2.4}}{\lfloor 0} = 0.091$$

Hence, the required probability is 0.091

EXERCISE 9.5

1. For the Poisson distribution, find
   (i) P(2), given $\lambda = 0.7$ (ii) P(3), given $\lambda = 2$.
2. Suppose a book of 614 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free of errors?
3. Experience shows that 1.4% of the telephone calls received are wrong numbers. Determine the probability that among 150 calls received, two are wrong numbers.
4. If 2% of the books bound at a certain workshop have defective binding, find the probability that 5 books out of 400 books will have defective binding.
5. It is given that 3% defective electric bulbs are manufactured by a company. Using Poisson distribution, find the probability of 100 bulbs will contain no defective bulb. (use $e^{-3} = 0.05$)
6. The mortality rate for a certain disease is 0.007. Using Poisson distribution, calculate the probability for 2 deaths in a group of 400 people.
7. A computer disk manufacturer tests disk quality on random basis before approving it. The approval is based on the number of errors in a test area on each disk and follow Poisson distribution with $\lambda = 0.2$. What is the percentage of test areas having two or a smaller number of errors?
8. A statistician records the number of trucks approaching a particular intersection to analyse the flow of traffic. He observes that on an average 1.6 trucks approach the intersection every minute. Assuming that the number of trucks approaching the intersection, follow a Poisson distribution model, what is the probability that 3 or more trucks will approach the intersection with in a minute?
9. The probability that a man aged 35 years will die before reaching the age of 40 years may be taken as 0.018. Out of a group of 400 men, now aged 35 years, what is the approximate probability that 2 men will die within the next 5 years? (Use $e^{-7.2} = 0.000747$).

10. Assume that the probability that a bomb dropped from an aeroplane will strike a certain target is $\frac{1}{5}$. If 6 bombs are dropped, find the probability that

(i) exactly two will strike the target (ii) atleast 2 will strike the target.

11. Ten percent of the bolts produced in a certain factory turn out to be defective. Find the probability, using Poisson distribution, that in a sample of 10 bolts chosen at random, (i) exactly 2 (ii) more than 2 bolts will be defective. (Take $e^{-1} = 0.368$)

12. At a busy traffic intersection, the probability p of an individual car having an accident is very small, say $p = 0.0001$. However, during a certain part of the day, a large number of cars, say 1000, pass through the intersection. Under these conditions, what is the probability of two or more accidents occurring during that period? (Use $e^{-0.1} = 0.9048$).

13. There are 500 boxes, each containing 1000 ballot papers for election. The chance that a ballot paper is defective is 0.002. Assuming Poisson distribution, find the number of boxes containing atmost two defective ballot papers. (Use $e^{-2} = 0.1353$).

14. Assuming that 200 misprints are distributed randomly throughout a book containing 500 pages, find the probability that a given page contains

(i) exactly 2 misprints (ii) 2 or more misprints.

15. A box contains 200 tickets, each bearing one of the numbers from 1 to 200. 20 tickets are drawn successively with replacement from the box. Find the probability that atmost 4 tickets bear numbers divisible by 20.

16. Find the probability that atmost 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective.

17. A manufacturer of screws knows that 4% of his product is defective. If he sells the screws in boxes of 100 and guarantees that not more than 5 screws will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?

[**Hint.** Required probability = Prob. (number of defectives > 5)].

18. The standard deviation of a Poisson variate X is $\sqrt{3}$. Find the probability that $X = 2$. (Given $e^{-3} = 0.0498$).

19. For a Poisson's distribution, $3P(X = 2) = P(X = 4)$. Find $P(X = 3)$. Take $e^{-6} = 0.00248$.

20. Mean of a Poisson distribution is 6.25. Find its variance and standard deviation.

21. If 1% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs, the number of defective bulbs will be 0, 1, 2, 3, 4, 5 respectively. Use recurrence relation of Poisson distribution. Also find the probability that (i) 3 or more (ii) between 1 and 3, and (iii) less than or equal to 2 bulbs will be defective.

[**Hint.** (ii) Required probability = P(1) + P(2) + P(3)]

22. It is known that 3% of plastic buckets manufactured in a factory are defective. Using the poisson distribution on a sample of 100 buckets, find the probability of

(i) Zero defective buckets

(ii) Atmost one bucket is defective (Use $e^{-3} = 0.049$)

Answers

1. (i) 0.1218 (ii) 0.18 2. 0.497 3. 0.269 4. 0.0928 5. 0.05
6. 0.239 7. 99.9% 8. 0.217 9. 0.01936
10. (i) 0.2167 (ii) 0.3378 11. (i) 0.184 (ii) 0.08 12. 0.0047 13. 338
14. (i) 0.0536 (ii) 0.062 15. 0.9967 16. 0.772 17. 0.228 18. 0.2241
19. 0.08928 20. Variance = 6.25, standard deviation = 2.5
21. 0.368, 0.368, 0.184, 0.0613, 0.0153, 0.0031 (i) 0.08 (ii) 0.6133 (iii) 0.92
22. (i) 0.049 (ii) 0.196